Preconditioning the Stage Equations of Implicit Runge Kutta Methods

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Abstract: When using implicit Runge-Kutta methods for solving parabolic PDEs, solving the stage equations is often the computational bottleneck, because the dimension of the stage equations is related to the spatial discretization and can thus become very large. The solution of the stage equations hence often requires the use of iterative solvers, whose convergence can be less than satisfactory. Using spectral analysis, we study the properties of two recently introduced preconditioners for the stage equations, and their dependence on the associated Butcher tableau of the Runge-Kutta method. We then try to optimize the Butcher tableau for the performance of the entire solution process, rather than only the order of convergence of the Runge-Kutta method. To do so requires to carefully balance the numerical stability of the Runge-Kutta method, its order of convergence, and also the convergence of the iterative solver for the stage equations. We illustrate our result on a simple test problem and then outline possible generalizations.